Lower Bounds for the Two-Echelon Vehicle Routing Problem

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Multi-Echelon Distribution Systems are broadly used in practice and have been much studied in the literature. Most of these studies address issues related to the movement of flows throughout the system from their origins to their final destinations. A recent trend is to also focus on the management of the vehicle fleets required to provide transportation among the different echelons of the system. The term Multi-Echelons Vehicle Routing Problems broadly covers such settings, where the delivery from one or more depots to customers is managed by routing and consolidating freight through intermediate depots called satellites.

In this work, we address the Two-Echelon Vehicle Routing Problem (2E-VRP), operating as follows:

• freight for different customers is available at a depot, where it is consolidated into 1st-level vehicles;
• each 1st-level vehicle travels to a subset of satellites and returns to the depot;
• at satellites, freight is transferred from 1st-level vehicles to 2nd-level vehicles;
• each 2nd-level vehicle travels from the satellites to a set of customers, and then returns to the same satellite.

Service at each level is provided by an homogeneous fleet. The goal is to serve customers while minimizing the total transportation cost of the two-echelon system, satisfying the capacity constraints of the vehicles. We consider a single depot and a fixed number of capacitated satellites. All customer demands are fixed and known in advance. All customer demands
must be satisfied. No time windows are assumed for deliveries and satellite loading/unloading operations.

In this paper, we present new lower bounds for the 2E-VRP. The adopted strategy consists in splitting the problem into easier subproblems, one for each level. Then, the lower bounds do not explicitly consider all the interactions between the two levels, in order to compute a global lower bound as the sum of lower bounds on sub-problems.

1 Literature review

A flow model for the 2E-VRP, with which instances up to 20 customers can be solved to the optimum, has been presented by Gonzalez Feliu, Perboli, Tadei and Vigo [5], while for what concern larger size instances, a fast cluster-based heuristic method has been proposed by Crainic, Mancini, Perboli, Tadei [2].

For an application of the 2E-VRP on freight distribution system we refer the reader to [3] in which advanced freight distribution systems are provided and in [4] in which the authors investigate the City Logistic Planning Problem, propose a general model and formulations for the main system components and identify promising solution avenue. On 50 customers instances, the gap between lower and upper bounds, obtained with the flow model, is high, and even if with the heuristic methods is possible to obtain much better upper bounds, the achieved gap is still quite big. For this reason, we decided to concentrate our efforts on the research of a tighter lower bound, even if we do not neglect the importance of the upper bounds quality.

2 Lower bounds for the 2E-VRP

Our goal is to find relevant bounds valid for the entire problem. The adopted strategy consists in splitting the problem into two easier subproblems. Since the provided methods are not considering at all the interaction between the two levels, it is possible to compute the global lower bound as the sum of local lower bounds, which can be computed more easily with respect to the full problem one. In fact, the problem of assigning customers to satellites can be incorporated in the first level problem; in this way the second level one could be treated as a standard multi depot VRP. First of all, we present bounds for the first level problem, and successively for the second level one.

2.1 First Level Bounds

Three strategies have been developed to compute first level bounds: The first adopted procedure for the calculation of a lower bound, called TSP, consists in operating a relaxation on the vehicle capacity in order to have the possibility to bring all the goods to the satellite using only one huge fictitious vehicle. Under this hypothesis the VRP problem becomes a TSP problem among the depot and all the satellites. This approximation is valid in the most cases, but not in all. For example, let consider an instance in which the optimum solution is obtained using only a subset of the available satellites; in this case the TSP on the whole satellites
could not be anymore a lower bound for the first level VRP. Nevertheless this procedure can be adopted on instances in which the total demand is satisfiable only using all the satellites, or when the vehicles capacity is such that, using every satellites subset, the covered distance is always inferior to the one covered in the best solution of the TSP.

The second one is the **Shortest Distance** procedure, in which the lower bound is computed by supposing, that, being known the minimum number of necessary vehicles for the first level, every vehicle starts from the depot, brings all the goods to the nearest satellite $S$ and comes straight back to the depot. This is the shortest distance each vehicle must cover so, that value, multiplied by the number of first level vehicles $V$, gives always a lower bound for the first level, even if, in the cases in which the capacity of $S$ is smaller than the total demand, this first level solution can never being part of a global feasible solution. In that case, the obtained lower bound could be not very tight.

The third procedure, **Shortest Distance with Capacity Constraints** is an improvement of the previous one. The difference is that in this case we are taking into consideration also the satellites capacity. The lower bound is computed by supposing, that, being known the minimum number of necessary vehicles for the first level, a demand equal to its capacity is assigned to the nearest satellite, and if there is yet not assigned demand, we start to assign it at the second nearest one, and so on, until the whole demand has been assigned. It is easy to prove that this bound is always larger or equal to the one obtained with the previous procedure and, more precisely, it is equal to it only if the nearest satellites capacity is larger than the total demand.

### 2.2 Second Level Bounds

The first adopted procedure for the calculation of a second level lower bound, the **TSP** one, consists in operating a relaxation on the vehicle capacity in order to have the possibility to bring all the goods to the satellite using only one huge vehicle. In this case, the lower bound computation could be split in two phases. First of all, we solve a TSP among all the customers, and after that we have to calculate the connection cost between satellites and customers. Let suppose to have already computed the number of necessary second level vehicles, $V$, we could calculate a lower bound assuming that the first vehicle starts its route from satellite $S'$ and visit, as first customer, customer $C'$ where $S'$ and $C'$ are chosen in order to minimize the distance $(s, c)$ computed on all $s$ in $S$, where $S$ is the satellites set, and on all $c$ in $C$, being $C$ the customers set. We also assume that the last visited customer $C''$ and the satellite in which the vehicle ends its route, $S''$, are computed in the same way, i.e. trying to find the couple $s, c$ that minimizes the distance $(s, c)$ computed on all satellites and on all customers except the already visited customer. This procedure can be applied for each vehicle, considering only the customers that have not been visited yet.

The **Macro-node** approach consists in treating the second level routing problem as a CVRP in which, the depot is represented by a macro-node including the real depot and all the satellites, and the distance between a customer and the macro-node is equal to the shortest distance between the considered customer and a satellite. If we solve the CVRP to the optimum, this value could be also used for computing very quickly an upper bound for the second level problem, considering the route of the optimal solution and assigning all the demands of the
customers included in each route, to the satellite which is nearest to the first customer of the route, if the assignment is feasible, i.e., if the satellite capacity constraint is respected, otherwise, assigning them to the second nearest, and so on, until we reach a feasible assignment. After that, the first level can be solved as a VRP problem in which we consider the satellites as customers with a demand equal to the sum of the demands of customers assigned to it.

The Multi-Depot approach consists in treating the second level routing problem as a Multi-Depot problem in which we are considering the satellites as depots. This approach allows us to use all the lower bounds for the Multi-Depot known in literature, which can give us a great accuracy of the bounds, but, furthermore, if we solve the problem to the optimum, the obtained results is still a local lower bound, but it is also a feasible solution for the second level. In this case we can start from it to build a feasible solution for the first level. In this way, the global solution obtained is feasible and can be considered as an upper bound. The Multi-Depot problem is solved by the branch-and-cut method developed by C.Contardo et al. [1]

3 Computational results

We have tested the methods on a set of 50-customer instances with different numbers of satellites, and various typologies of customers and satellites distributions. Figure 1 presents a summary of the computational results. In column 1 and 2 we report the lower bound used for the first and the second level, respectively. Column 3 shows the mean of the gaps between the best solution and the lower bound on all instances. Finally, column 4 presents the best percentage gap between the best solution and the lower bound on the instances. The best solution was obtained as the best among the literature and the solution that can be derived solving the multi-depot VRP to optimality.

Regarding the first level lower bounds, the best results are obtained with the Shortest Distance with capacity constraints method. TSP works properly when the number of total freight is not tight to the fleet capacity. In the other cases, the relaxation on the capacity constraint does not allow us to reach a very tight bound. The same can be said relative to the TSP bounds for the second level, while both Macro-node and Multi-Depot obtain much better results.

4 Conclusions

In this paper we presented new lower bounds for the 2E-VRP. In the presentation we will discuss more in details the computational issues, as well as the impact of the system layout on the gaps and how to use them to compute new upper bounds for the problem.
Lower Bounds for 2E-VRP

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Figure 1: Mean and best values of Lower Bounds

References


