A Column Generation Based Heuristic for the Capacitated Location-Routing Problem

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1 Introduction

As pointed out by Nagy and Salhi in their recent survey [2], many variants of the location-routing problem (LRP) are addressed in the literature. Generally the LRP consists of determining locations for depots from which customers are served on routes with the objective of minimizing the overall cost. In this talk we consider the capacitated location-routing problem (CLRP) which is defined as follows. Let \( G = (X, A) \) be an undirected graph where \( X \) is the vertex set and \( A \) the edge set. \( X = I \cup F \) is composed of \( n \) customers \( i \) in \( I \) and \( m \) potential facilities \( f \) in \( F \). A cost matrix is defined on \( A \) and a fixed opening cost is associated with each vertex of \( F \). Each customer \( i \) must be served a demand \( d_i \) from a depot. The total demand served from one depot \( f \) must not exceed the depot capacity \( K_f \). To deliver the demand, a fleet of vehicles is available and with each vehicle is associated a maximal capacity \( Q \). A solution of the CLRP is a set of location sites for the depots and a collection of routes where: (i) each customer is visited only once; (ii) the total demand for each route is at most \( Q \); and (iii) the total demand delivered from each depot \( f \) is at most \( K_f \). The CLRP aims to determine a minimal total cost solution. The total cost is the sum of the opening costs and of the routing costs. Its solution provides sets of customers associated with routes and sets of opened depots.

In this work we propose an hybrid heuristic based on a column generation scheme where the subproblem is solved approximately using a tabu search algorithm. Computational results obtained by the described algorithm compete favorably with the most efficient heuristic previously described in the literature [3].
2 A set partitioning model for the CLRP

To describe the model on which our approach is based, we introduce the following additional notation:

\( R \) : set of clusters of customers,
\( p \) : size of the fleet,
\( D_r \) : total demand of cluster \( r \in R \),
\( C_f \) : fixed cost of opening facility \( f \),
\( c_{ij} \) : cost of traversing arc \((i, j)\),
\( q \) : upper bound on the number of opened facilities,
\( f_{ri} = 1 \) if customer \( i \) belongs to cluster \( r \), 0 otherwise,
\( \tilde{c}_{fr} \) : cost of serving the customers of cluster \( r \) from facility \( f \).

We define the following decisions variables:

\[
  z_{fr} = \begin{cases} 
    1 & \text{if the customers of cluster } r \text{ are served from facility } f \\
    0 & \text{otherwise},
  \end{cases}
\]

\[
  z_f = \begin{cases} 
    1 & \text{if facility } f \text{ is opened} \\
    0 & \text{otherwise}.
  \end{cases}
\]

Then the model \((P)\) is as follows:

\[
  \min \quad z = \sum_{f=1}^{m} C_f z_f + \sum_{f=1}^{m} \tilde{c}_{fr} z_{fr} \quad (1)
\]

subject to

\[
  \sum_{f=1}^{m} \sum_{r \in R} f_{ri} z_{fr} \geq 1 \quad i \in I \quad (2)
\]

\[
  \sum_{r \in R} z_{fr} \leq p z_f \quad f \in F \quad (3)
\]

\[
  \sum_{f=1}^{m} z_f \leq q \quad (4)
\]

\[
  \sum_{r \in R} D_r z_{fr} \leq K_f z_f \quad f \in F \quad (5)
\]

\[
  z_{fr} \in \{0, 1\} \quad (6)
\]

\[
  z_f \in \{0, 1\} \quad (7)
\]
Equation (1) defines the objective function. The total cost is computed as the sum of the fixed cost for opened facilities and of the routing cost for the selected clusters. Inequalities (2) ensure that each customer is visited at least once. Constraints (3) ensure that routes are initiated from a facility if and only if the facility is opened. Constraint (4) restricts the number of opened facilities. Restrictions (5) are the capacity constraints on the facilities. Finally, constraints (6) and (7) define the type of decision variables.

3 Column generation heuristic

In this section we describe an hybrid heuristic to solve the CLRP. This heuristic is based on a column generation approach.

In the initialization step, we solve a generalized capacitated concentrator location problem. A solution of this problem is composed of several clusters of customers. The customers belonging to a given cluster are served on the same feasible route. We consider a subset of set $\bar{R} : \bar{R}$ which is initialized with these clusters. In the course of the algorithm, $\bar{R}$ will increase since we will add new clusters as soon as feasible routes will be identified.

In Step 2, we consider the linear relaxation of the model $(P)$ described in Section 2. This linear program constitutes the master problem in our column generation approach. As usual we do not solve the master problem but the so-called restricted master problem $(RPM)$ in which the set of clusters $R$ is substituted by $\bar{R} \subseteq R$. This means that only a subset of clusters is considered which does not guarantee that the solution of the $RPM$ is optimal with respect to the master problem.

Step 3 aims to check whether or not the current solution of the $RPM$ is optimal. In other words, if there exists a cluster $r$ of customers such that the reduced cost of variable $z_{fr}$ is negative, the current solution is not optimal. To identify such clusters we have to solve the subproblem which can be modeled as a Multi-Depot Vehicle Routing Problem (MDVRP). Let $f'$ be one of the depots. Initially, $f'$ is set to 1. In order to build new clusters with negative reduced costs, we modify the initial cost matrix by considering dual variables $\alpha_i$, $\beta_{f'}$, $\gamma_i$, and $\delta_f$ associated respectively to constraints (2), (3), (4) and (5). For all $i,j \in I$, new costs are equals to $c'_{ij} = (c_{ij} - \alpha_i - \delta_{f'}d_i)$ since the reduced cost of $z_{f'r}$ can be computed as $\tilde{c}_{f'r} = \sum_{i,j} (c_{ij} - \alpha_i - \delta_{f'}d_i) x_{ij} - \beta_{f'}$. The subproblem is not solved exactly but heuristically thanks to a tabu search algorithm originally proposed by Cordeau, Gendreau and Laporte [1]. If the total cost of a computed route is lower than $\beta_{f'}$ then we add the associated cluster of customers to $\bar{R}$ and go to Step 2 otherwise we increase $f'$ and reiterate this step. The algorithm stops as soon as we cannot identify a route with a negative reduced cost for $f' = m$.

Finally the last step consists in solving the initial model $(P)$ with the subset of clusters $\bar{R}$ generated previously. This integer program is solved using a standard commercial solver.
4 Computational results

The algorithm just described was coded in C++ and run to a 3.4 GHz Pentium IV Extreme Edition. Integer and linear programs were solved with CPLEX 9.1. To assess the efficiency of the proposed heuristic, we considered 30 instances generated by Prins et al. [3] for the CLRP. Computational results are summarized in Table 1. The table headings are as follows:

\( n \) : number of customers,
\( m \) : number of potential facilities,
LRGTS : heuristic algorithm proposed by Prins et al. for the CLRP,
Column Generation : our heuristic for the CLRP,

For each approach, CPU and costs are given.

<table>
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<th>LRGTS</th>
<th>Column Generation</th>
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<td></td>
<td>(Prins, Prodhon, Ruiz, Soriano, Wolfler-Calvo)</td>
<td>(Boulanger, Semet)</td>
</tr>
<tr>
<td>( n )</td>
<td>( m )</td>
<td>cost</td>
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Results reported in Table 1 are averages for each class of instances characterized by the number of customers and the number of potential facilities. As it can be observed, none of the methods outperforms the other even if our method consumes more time. The proposed heuristic obtains new best known solutions for six instances and leads to solutions within 3.5 percents of the best known solution values.

5 Conclusions

We have proposed a set-partitioning model for the CLRP and have described a new hybrid heuristic approach based on a column generation scheme. This approach was compared to the heuristic developed by Prins, Prodhon, Ruiz, Soriano, and Wolfler Calvo and leads to competitive results.
References

